Variance explained by different model components does not behave like a Venn diagram: Why variance decomposition provides misleading intuitions

Western BrainsCAN Transforming brain research.

J. Derrick Xiang, Marieke Mur*, Jörn Diedrichsen*
* co-senior authorship
Western Institute for Neuroscience, University of Western Ontario, Canada

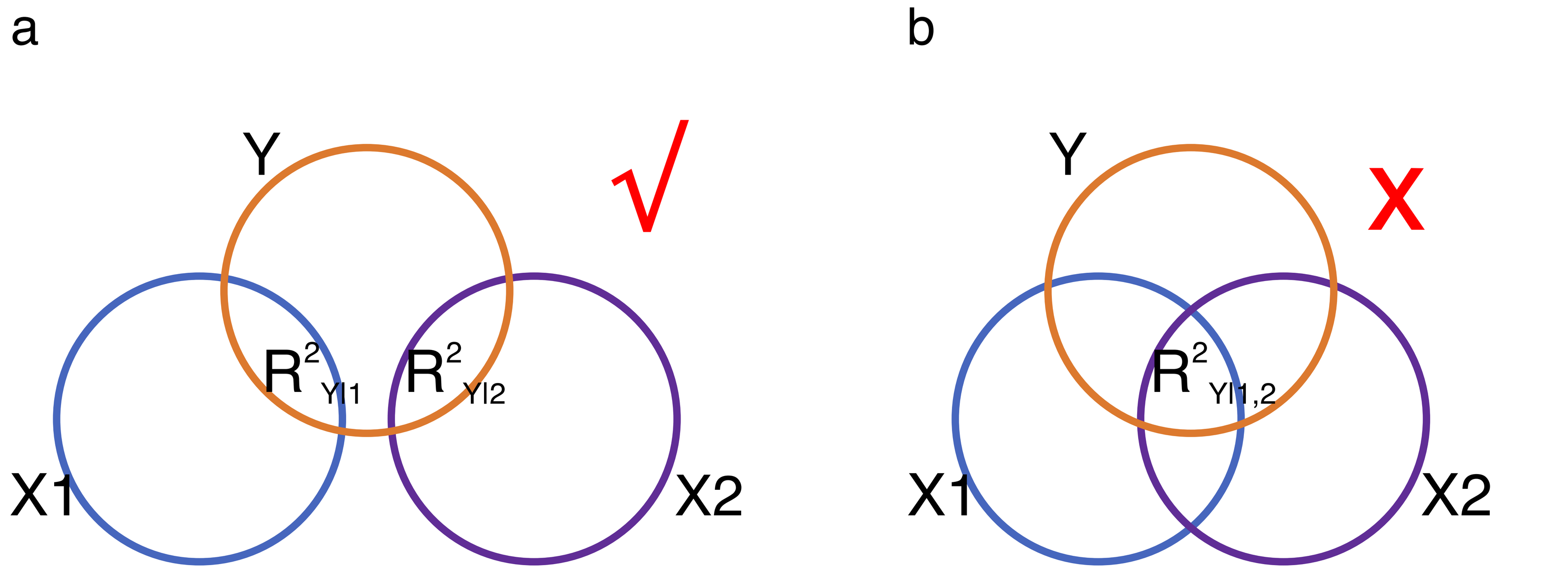
Western Institute for Neuroscience

Introduction

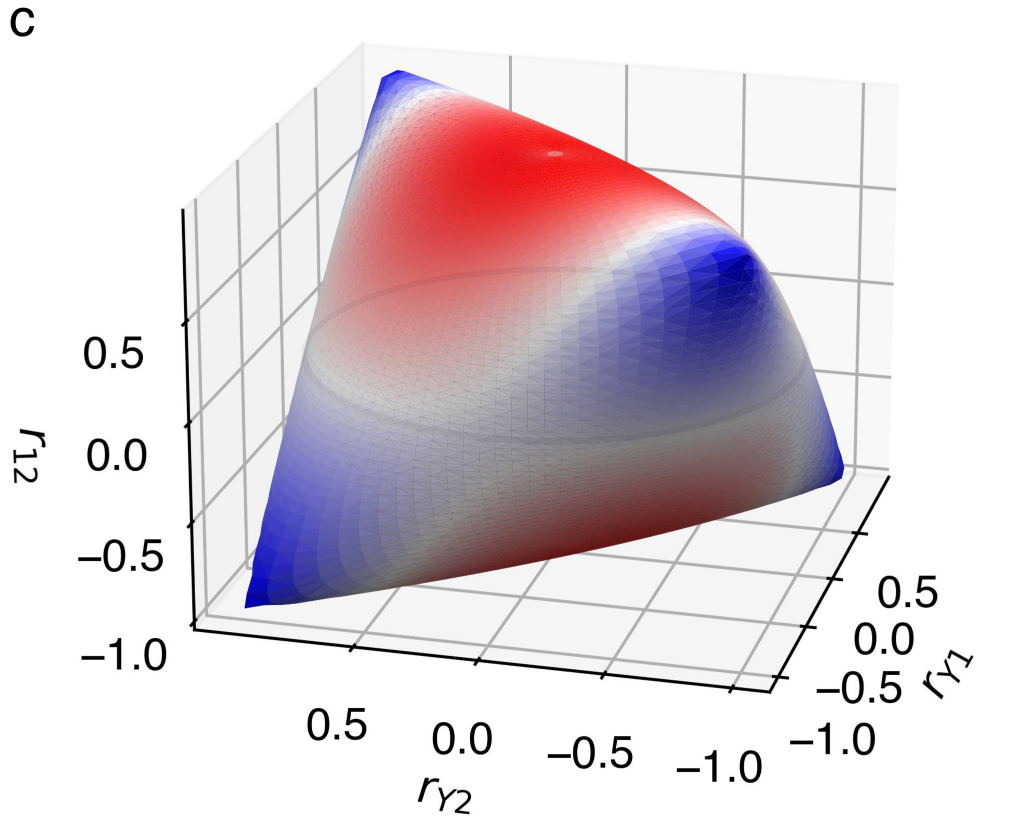
- Researchers often rely on the idea of variance decomposition when interpreting the contributions of multiple predictors to explaining brain responses[1-4].
- The underlying intuition -- rooted in Fisher's analysis of variance (ANOVA), where predictors X1 and X2 are orthogonal, the variance explained by the two together equals the sum of the variance explained by each one alone: $R^2_{Y|1U2}=R^2_{Y|1}+R^2_{Y|2}$[5].
- Several studies have sought to extend this logic to the more common case of correlated predictors. While intuitively appealing, this approach can lead to incorrect conclusions about how models with multiple correlated predictors behave.
- We propose the Model Family framework as an alternative to quantitatively assess the contribution of predictors. This work applies to commonly used regression-based approaches, such as linear regression, RSA, PCM and encoding models[6,7].

Methods

(1) Variance decomposition breaks down for correlated predictors



c



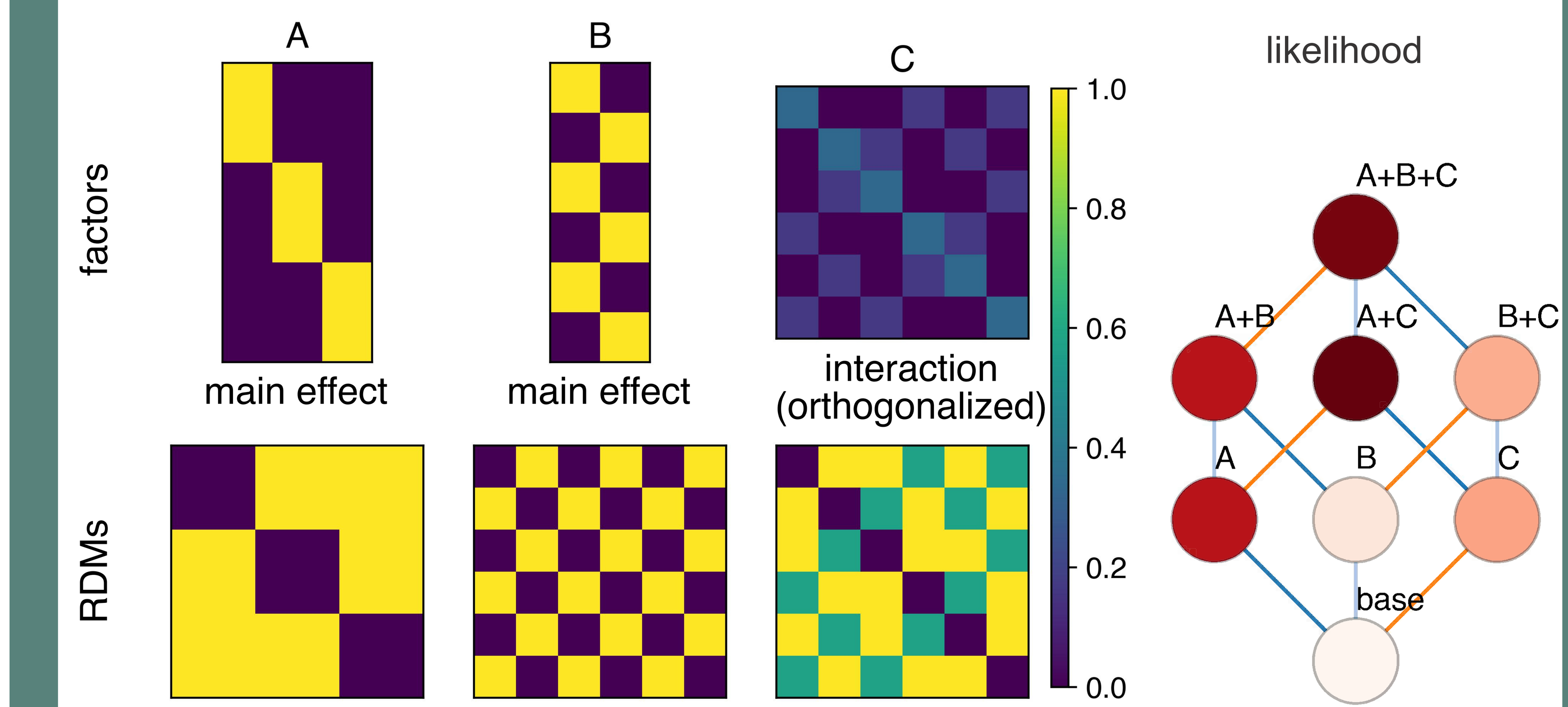
Correlations among variables:

	Y	X1	X2
Y	1	r_{Y1}	r_{Y2}
X1	r_{Y1}	1	r_{12}
X2	r_{Y2}	r_{12}	1

(a) In the case of uncorrelated predictors, $R^2_{Y|1U2} = R^2_{Y|1} + R^2_{Y|2}$
(b) In the case of correlated predictors, $R^2_{Y|1,2} = R^2_{Y|1} + R^2_{Y|2} - R^2_{Y|1U2}$, could be negative, leading to uninterpretable results.
(c) $R^2_{Y|1,2} < 0$ (suppression) happens half of the time. For all possible combinations of r_{Y1}, r_{Y2} and r_{12}, suppression occurs in the red regions but not the blue regions.

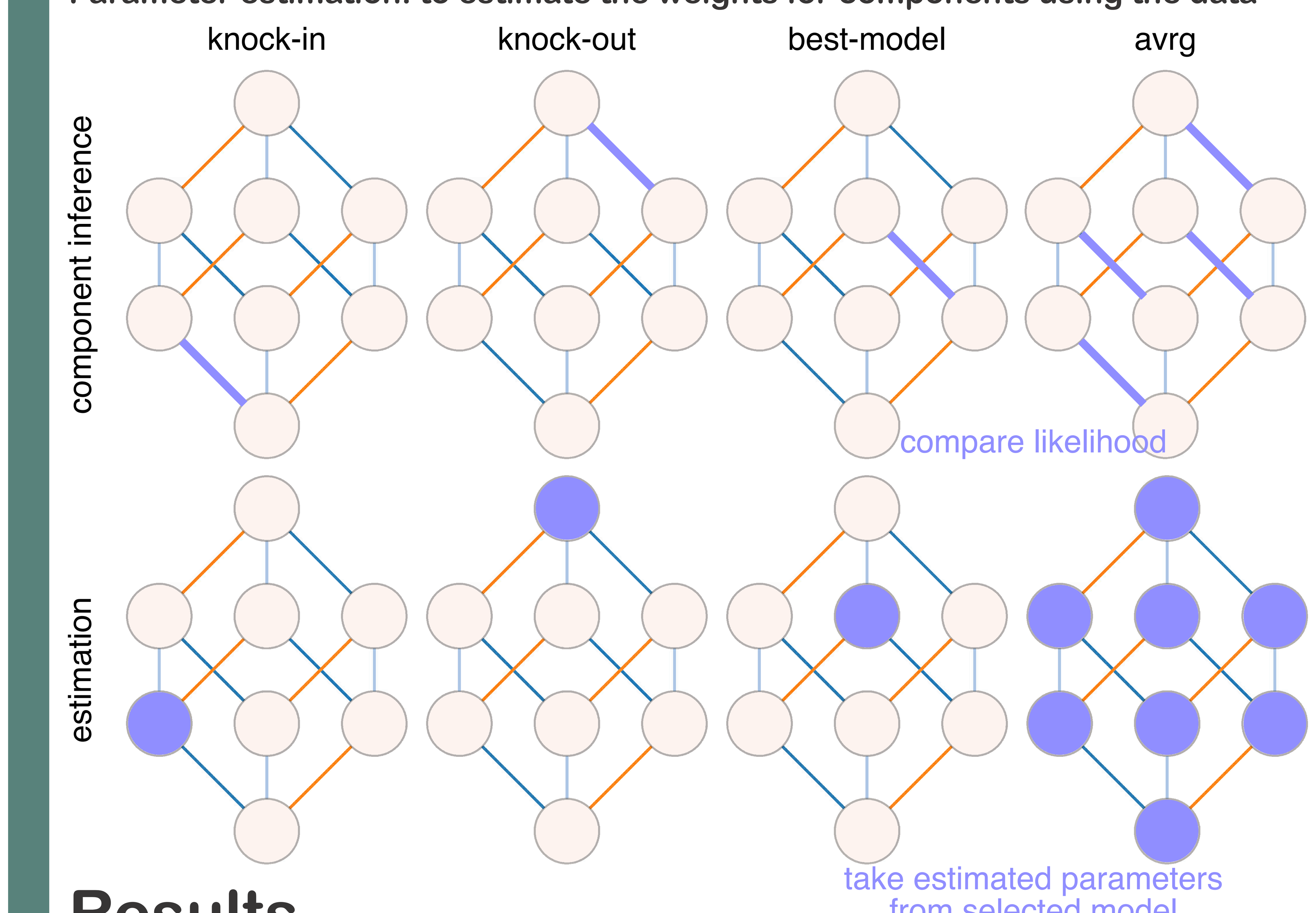
Variance is not a fixed quantity of the data that can be decomposed, but should be considered in the context of all model components -- particularly when interpreting model performance or comparing models.

(2) Model Family



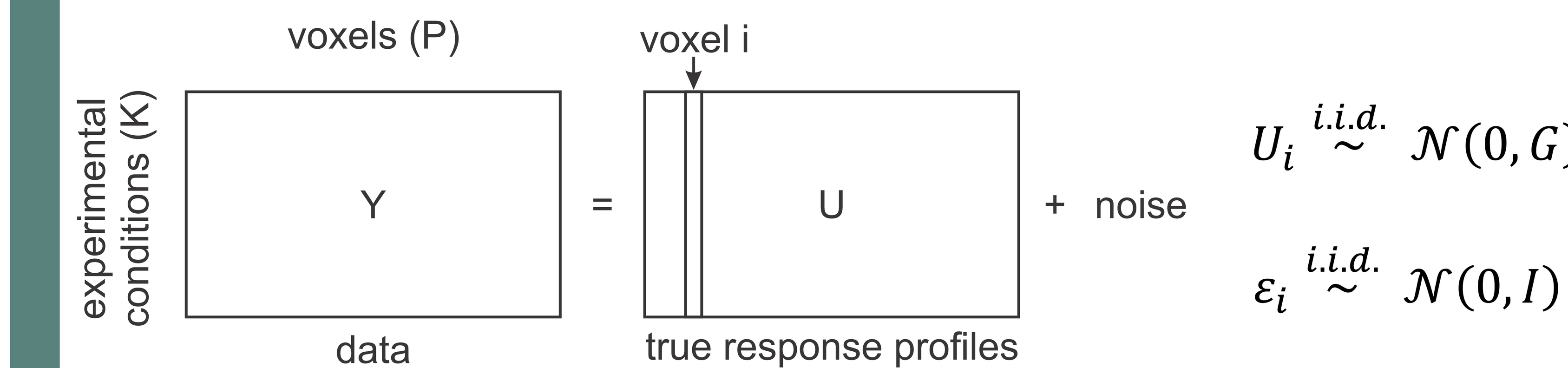
(3) Methods for component inference and parameter estimation

Component inference: to infer whether a component is present in the brain data
Parameter estimation: to estimate the weights for components using the data

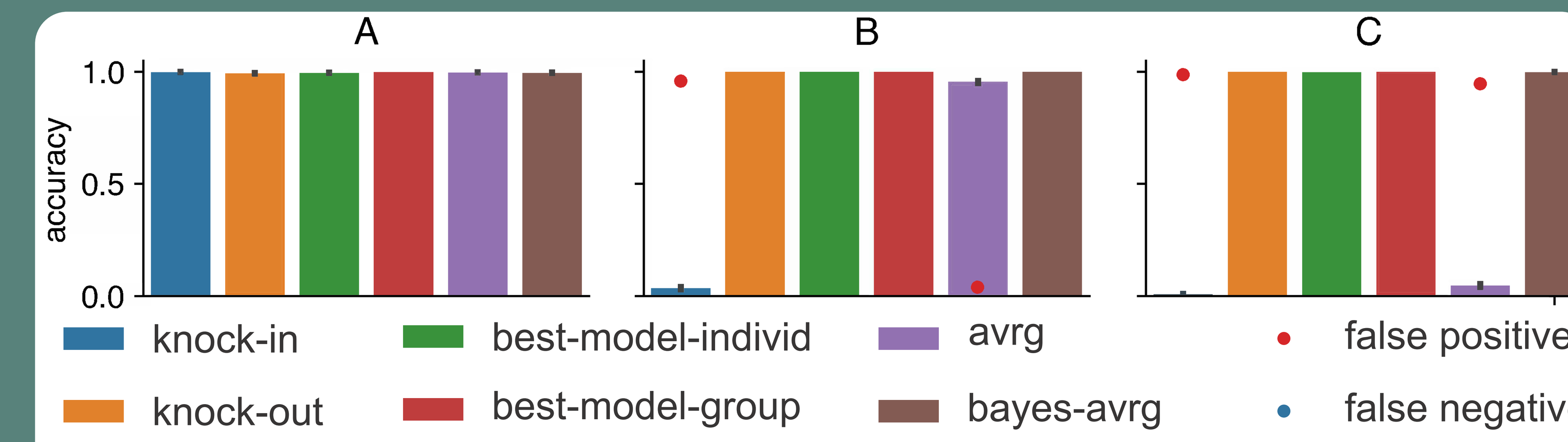


Results

(1) Knock-in and avrg could fail component inference



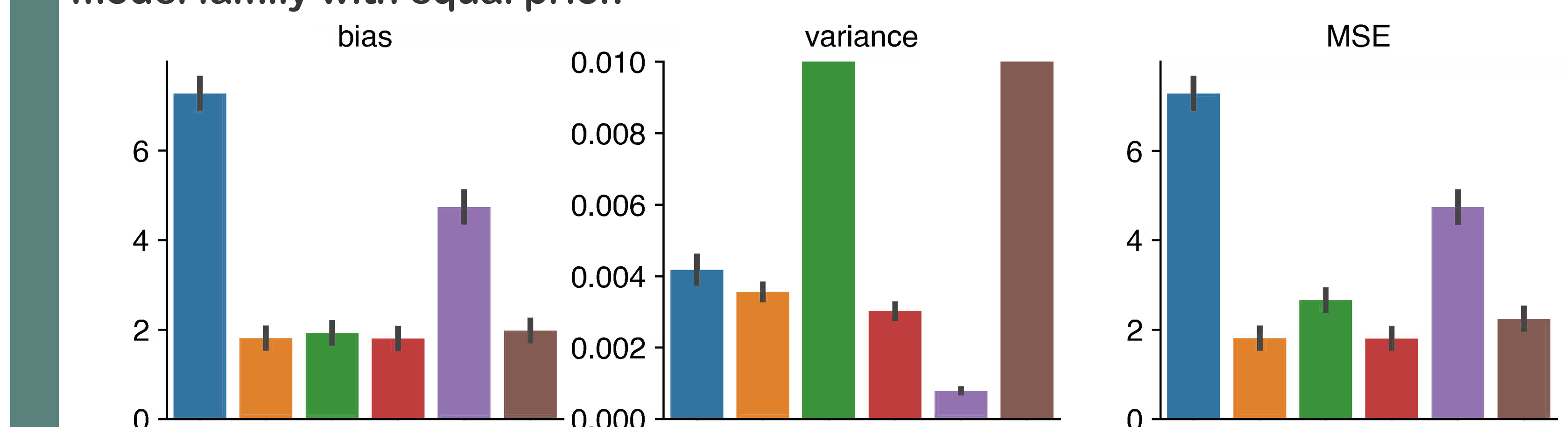
Simulation using RSA: each simulation contains multi-voxel data for 20 subjects across 8 runs, 200 simulations in total. Only A is present in the data.
Runs for a subject only differ by noise; across subjects, true response profiles differ but share the G.



Knock-in and avrg fail component inference due to false positive.

(2) Knock-in and avrg could fail estimation

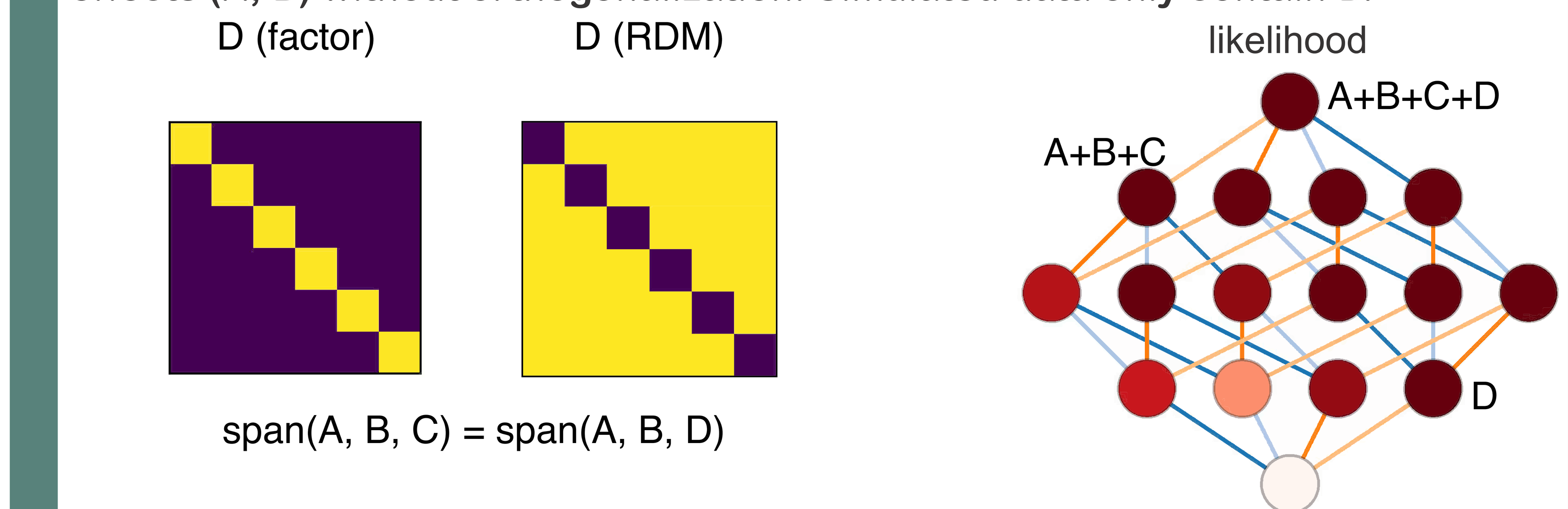
Following the previous example, the data are generated by each model in the model family with equal prior.

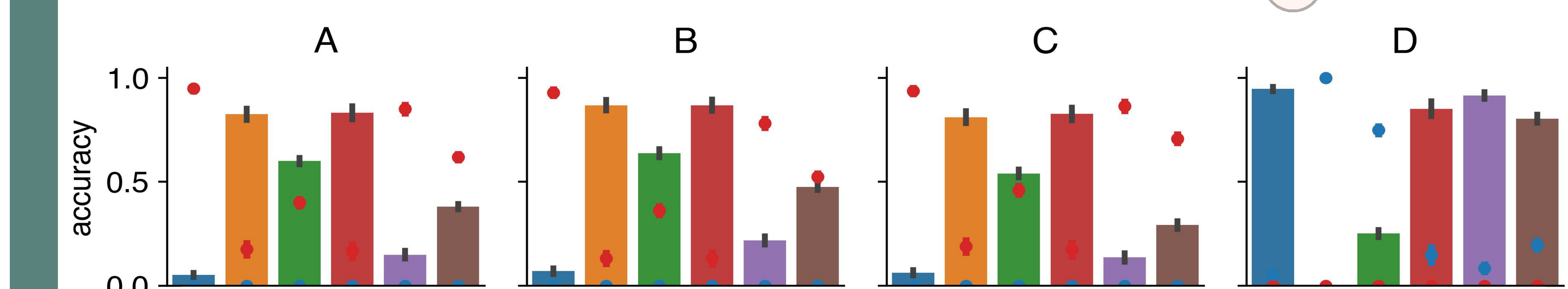


Knock-in and avrg lead to high bias for parameter estimation.

(3) Knock-out could fail component inference

Besides components A, B, C, now include D, which is the interaction of main effects (A, B) without orthogonalization. Simulated data only contain D.





Knock-out fails due to false negative; knock-in due to false positive.

Summary and conclusion

- Variance decomposition does not work for correlated predictors. We propose Model Family framework as an alternative.
- Out of the methods to choose from for component inference and parameter estimation, knock-in, knock-out, avrg could fail. Best-model-individual tends to yield high variance.
- Best-model-group and bayes-avrg are more reliable methods.

References

1 Lescroart et al (2015) Frontiers in Comp Neuro. 2 Heer et al (2017) JNS. 3 Bonner et al (2018) PLOS Comp Bio. 4 Huth et al (2012) Neuron. 5 Fisher (1925) Statistical methods. 6 Diedrichsen et al (2017) PLOS Comp Bio. 7 Kriegeskorte et al (2008) Frontiers in Systems Neuro.